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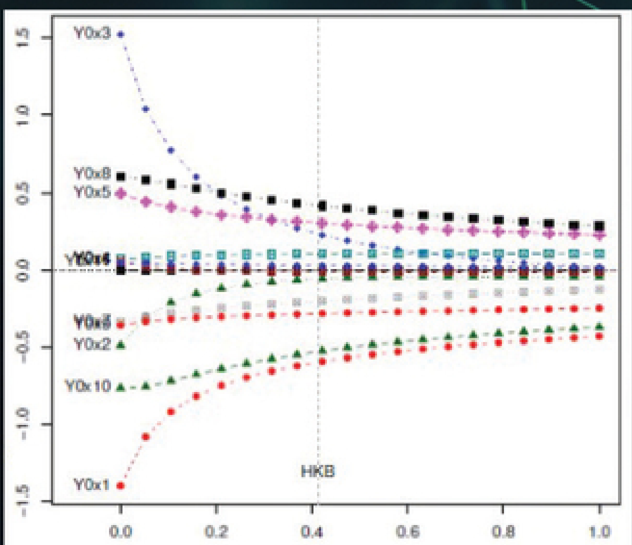
Theory of Ridge Regression Estimators with Applications

A.K. Md. Ehsanes Saleh | Mohammad Arashi
B.M. Golam Kibria



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To our wives
Shahidara Saleh
Reihaneh Arashi
Farhana Kibria
(Orchi)

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Preface

Regression analysis is the most useful statistical technique for analyzing multifaceted data in numerous fields of science, engineering, and social sciences. The estimation of regression parameters is a major concern for researchers and practitioners alike. It is well known that the least-squares estimators (LSEs) are popular for linear models because they are unbiased with minimum variance characteristics. But data analysts point out some deficiencies of the LSE with respect to prediction accuracy and interpretation. Further, the LSE may not exist if the design matrix is singular. Hoerl and Kennard (1970) introduced "ridge regression," which opened the door for "penalty estimators" based on the Tikhonov (1963) regularization. This methodology is the minimization of the least squares subject to an L_2 penalty. This methodology now impacts the development of data analysis for low- and high-dimensional cases, as well as applications of neural networks and big data analytics. However, this procedure does not produce a sparse solution. Toward this end, Tibshirani (1996) proposed the least absolute shrinkage and selection operator (LASSO) to overcome the deficiencies of LSE such as prediction and interpretation of the reduced model. LASSO is applicable in high- and low-dimensional cases as well as in big data analysis. LASSO simultaneously estimates and selects the parameters of a given model. This methodology minimizes the least squares criteria subject to an L_1 penalty, retaining the good properties of "subset selection" and "ridge regression."

There are many other shrinkage estimators in the literature such as the preliminary test and Stein-type estimators originally developed by Bancroft (1944), and Stein (1956), and James and Stein (1961), respectively. They do not select coefficients but only shrink them toward a predecided target value. There is extensive literature on a parametric approach to preliminary test and Stein-type estimators. The topic has been expanded toward robust rank-based, M-based, and quantile-based preliminary test and Stein-type estimation of regression coefficients by Saleh and Sen (1978–1985) and Sen and Saleh (1979, 1985, 1987). There is extensive literature focused only on this topic, and most recently documented by Saleh (2006). Due to the immense impact of Stein's approach on point estimation, scores of technical papers appeared in various areas of application.

The objective of this book is to provide a clear and balanced introduction of the theory of ridge regression, LASSO, preliminary test, and Stein-type estimators for graduate students and research-oriented statisticians, postdoctoral, and researchers. We start with the simplest models like the location model, simple linear model, and analysis of variance (ANOVA). Then we introduce the seemingly unrelated simple linear models. Next, we consider multiple regression, logistic regression, robust ridge regression, and high dimensional models. And, finally, as applications, we consider neural networks and big data to demonstrate the importance of ridge and logistic regression in these applications.

This book has 12 chapters, according to the given description of materials covered. Chapter 1 presents an introduction to ridge regression and different aspects of it, stressing the multicollinearity problem and its application to high-dimensional problems. Chapter 2 considers the simple linear model and location model, and provides theoretical developments of it. Chapters 3 and 4 deal with the ANOVA model and the seemingly unrelated simple linear models, respectively. Chapter 5 considers ridge regression and LASSO for multiple regression together with preliminary test and Stein-type estimators and a comparison thereof when the design matrix is nonorthogonal. Chapter 6 considers the ridge regression estimator and its relation with LASSO. Further, we study the properties of the preliminary test and Stein-type estimators with low dimension in detail. In Chapter 7, we cover the partially linear model and the properties of LASSO, ridge, preliminary test, and the Stein-type estimators. Chapter 8 contains the discussion of the logistic regression model and the related estimators of diverse kinds as described before in other chapters. Chapter 9 discusses the multiple regression model with autoregressive errors. In Chapter 10, we provide a comparative study of LASSO, ridge, preliminary test, and Stein-type estimators using rank-based theory. In Chapter 11, we discuss the estimation of parameters of a regression model with high dimensions. Finally, we conclude the book with Chapter 12 to illustrate recent applications of ridge, LASSO, and logistic regression to neural networks and big data analysis.

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